Saturday, 27 April 2002 0900 h — 1100 h

The Chinese High School
Mathematics Learning And Research Centre

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Asia Pacific Mathematical Olympiad for Primary Schools 2002

First Round
2 hours
(150 marks)

Instructions to Participants

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Write your answers in the answer boxes.
Marks are awarded for correct answers only.

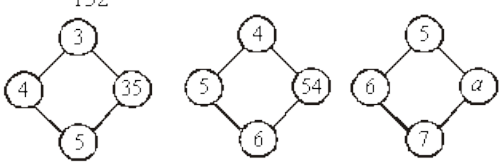
This question paper consists of 4 printed pages (including this page)

Number of correct answers for Q1 to Q10 : ______ Marks (´ **4**) : ______
Number of correct answers for Q11 to Q20 : ______ Marks (´ **5**) : ______
Number of correct answers for Q20 to Q30 : ______ Marks (´ **6**) : ______

1. How many numbers are there in the following number sequence ?
   1.11, 1.12, 1.13, ... , 9.98, 9.99.

2. What is the missing number in the following number sequence ?
   $\frac{1}{2}, \frac{1}{12}, \frac{1}{30}, \frac{1}{56}, \square, \frac{1}{132}$.

3. Observe the pattern and find the value of a.

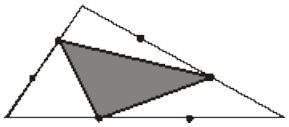


4. Find the value of

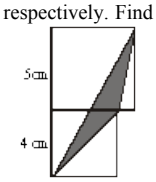
$$\frac{1}{2002} + \frac{2}{2002} + \frac{3}{2002} + \ldots + \frac{2000}{2002} + \frac{2001}{2002}.$$

5. The average of 10 consecutive odd numbers is 100. What is the greatest number among the 10 numbers ?

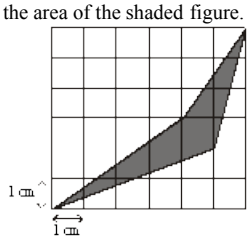
6. What fraction of the figure is shaded , when each side of the triangle is divided into 3 equal parts by the points?



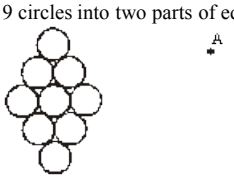
7. The figure is made up of two squares of sides 5 cm and 4 cm respectively. Find the shaded area.



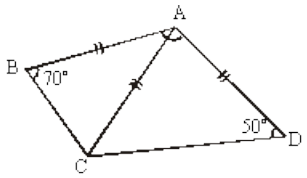
8. Find the area of the shaded figure.



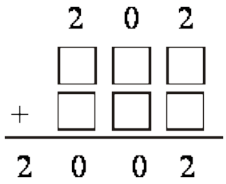
9. Draw a straight line through the point A to divide the 9 circles into two parts of equal areas.



10 In the figure, $AB = AC = AD$, $\angle ABC = 70^0$ and $\angle ADC = 50^0$.
Find $\angle BAD$.



11. In the sum, each $\square$ represents a non-zero digit. What is the sum of all the 6 missing digits ?



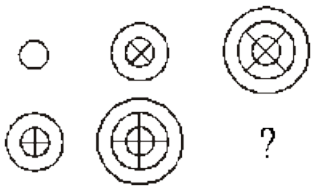
12. The average of n whole numbers is 80. One of the numbers is 100. After removing the number 100, the average of the remaining numbers is 78. Find the value of n.

13. The list price of an article is \$6000. If it is sold at half price, the profit is 25%. At what price must it be sold so that the profit will be 50% ?

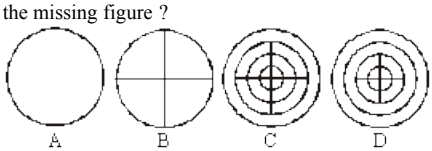
14. $\frac{1}{7}$ of a group of pupils score A for Mathematics; $\frac{1}{3}$ of the pupils score B; $\frac{1}{2}$ of the pupils score C; and the rest score D.
If a total of 100 pupils score A or B, how many pupils score D ?

15. At 8.00 a.m., car A leaves Town P and travels along an expressway. After some time, car B leaves Town P and travels along the same expressway. The two cars meet at 9.00 a.m. If the ratio of A's speed to B's speed is 4 : 5 , what time does B leave Town P ?

16.



Which one of the following is the missing figure ?

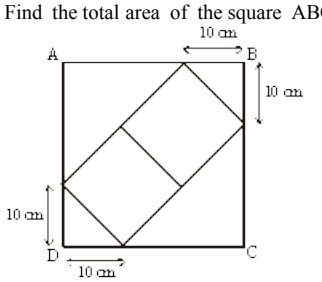


17. A rectangle is folded along a diagonal as shown.

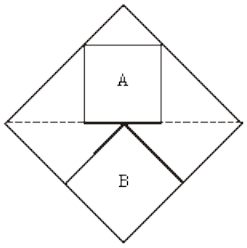
The area of the resulting figure is $\frac{5}{8}$ of the area

of the original rectangle. If the area of the shaded triangle is $18\ cm^2$, find the area of the original rectangle.

18. The square, ABCD is made up of 4 triangles and 2 smaller squares.
Find the total area of the square ABCD.



19. The diagram shows two squares A and B inside a bigger square.
Find the ratio of the area of A to the area of B.



20. There are 3 straight lines and 2 circles on the plane. They divide the plane into regions. Find the greatest possible number of regions.

21. The number 20022002 . . . 20022002 is formed by writing 2002 blocks of '2002'.
Find the remainder when the number is divided by 9.

22. Find the sum of the first 100 numbers in the following number sequence .
1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, . . .

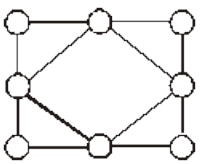
23. In a number sequence : 1, 1, 2, 3, 5, 8, 13, 21, . . . , starting from the third number, each number is the sum of the two numbers that come just before it.
How many **even** numbers are there among the first 1000 numbers in the number sequence ?

24 10 years ago, the ratio of John's age to Peter's age was 5 : 2.
The ratio is 5 : 3 now. What will be the ratio 10 years later ?

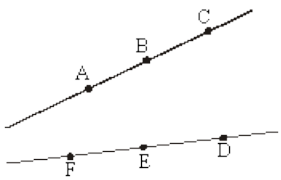
25. David had \$100 more than Allen at first. After David's money had decreased by \$120 and Allen's money had increased by \$200, Allen had 3 times as much money as David.
What was the total amount of money they had at first ?

26. Two barrels X and Y contained different amounts of oil at first.
Some oil from X was poured to Y so that the amount of oil in Y was doubled. Then, some oil from Y was poured to X so that the amount of oil in X was doubled
After these two pourings, the barrels each contained 18 litres of oil. How many litres of oil were in X at first ?

27. In the figure, each circle is to be coloured by one of the colours : red, yellow and blue.
In how many ways can we colour the 8 circles such that any two circles which are joined by a straight line have different colours ?



28. The points A, B, C, D, E and F are on the two straight lines as shown.
How many triangles can be formed with any 3 of the 6 points as vertices ?



29. Patrick had a sum of money.

On the first day, he spent $\frac{1}{4}$ of his money and donated \$30 to charity.

On the second day, he spent $\frac{1}{3}$ of the money he still had and donated \$20 to charity.

On the third day, he spent $\frac{1}{2}$ of the money he still had and donated \$10 to charity.
At the end, he had \$10 left. How much money did he have at first ?

30. Four football teams A, B, C and D are in the same group. Each team plays 3 matches, one with each of the other 3 teams.

The winner of each match scores 3 points; the loser scores 0 points; and if a match is a draw, each team scores 1 point.

After all the matches, the results are as follows :

(1) The total scores of 3 matches for the four teams are consecutive odd numbers.

(2) D has the highest total score.

(3) A has exactly 2 draws, one of which is the match with C.

Find the total score for each team..

THE END

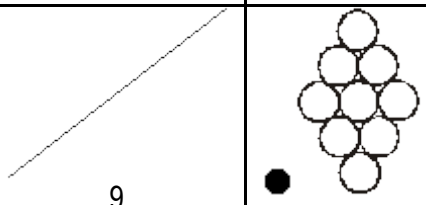
Name of Participant : _____ Index No : _____ / ____

(Statutory Name)

Name of School : _____

Singapore Mathematical Olympiad for Primary Schools 2002

First Round – Answers Sheet

	Answers	For markers' use only
1	889	
2	1/90	
3	77	
4	1000 ½	
5	109	
6	1/3	
7	8 cm²	
8	6 cm²	
9		Line must pass through the centre of the middle circle.
10	120°	

	Answers	For markers' use only
16	C	
17	48 cm²	
18	900 cm²	
19	8 : 9	
20	21	
Questions 11 to 20 each carries 5 marks		
21	7	
22	365	
23	333	
24	10 : 7	
25	$360	

Questions 1 to 10 each carries 4 marks		
11	36	
12	11	
13	$3600	
14	5	
15	8.12a.m.	

26	22.5 l	
27	18	
28	18	
29	$160	
30	A : 5 B : 3 C : 1 D : 7	All correct – 6m 3 correct – 2m Others – 0m
Questions 21 to 30 each carries 6 marks		

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Mathematics Learning And Research Centre

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Asia Pacific Mathematical Olympiad for Primary Schools 2002

Invitation Round
2 hours
(60 marks)

Instructions to Participants

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Working ***must*** be clearly shown in the space below each question.
Marks are awarded for both method and answer.
Each question carries 10 marks.

This question paper consists of 7 printed pages (including this page)

Question	1	2	3	4	5	6
Marks						

Total Marks for **Invitation Round** : _____

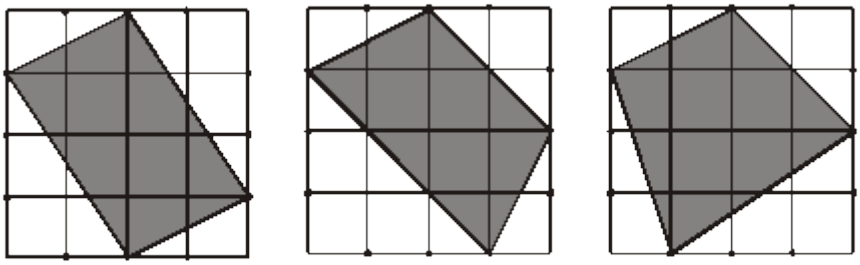
1. The following is an incomplete 9 by 9 multiplication table.

×	1	2	3	4	5	6	7	8	9
1				:			:		
2				:			:		
3				:			:		
4	. .	. .	. .	16			:		
5	. .	. .	. .	. .	. .	. .	35		
6									
7									
8									
9									

(a) Find out how many of the 81 products are odd numbers .

(b) If the multiplication table is extended up to 99 by 99, how many of the products are odd numbers ?

2. Find the area of each of the following shaded regions.



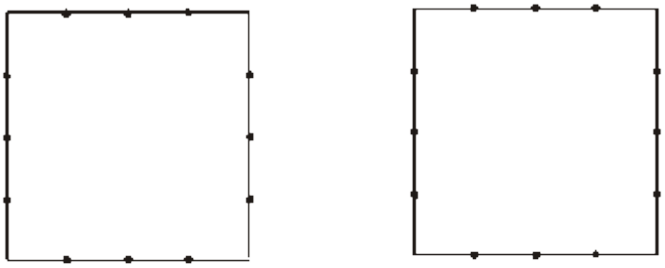
(A) (B) (C)

The shaded 4-sided figures above have been drawn with the four vertices at the dots, on each side of the square.

In the same manner,

(i) draw a 4-sided figure with the greatest possible area in (**D**),

(ii) draw a 4-sided figure with the smallest possible area in (**E**).



(D) (E)

4. There are two identical bottles A and B.

A contains $\frac{1}{2}$ bottle of pure honey.

B contains a full bottle of water.

First pour the water from B to fill up A and mix the content completely ;

then pour the mixture from A to fill up B and mix the content completely.

(i) What is the ratio of honey to water in B after the two pourings ?

(ii) If this process of pouring from A to B , and then from B to A, is repeated for another time, what will be the ratio of honey to water in B ?

(iii) If this process of pouring is repeated indefinitely, what will be the ratio of honey to water in B ?

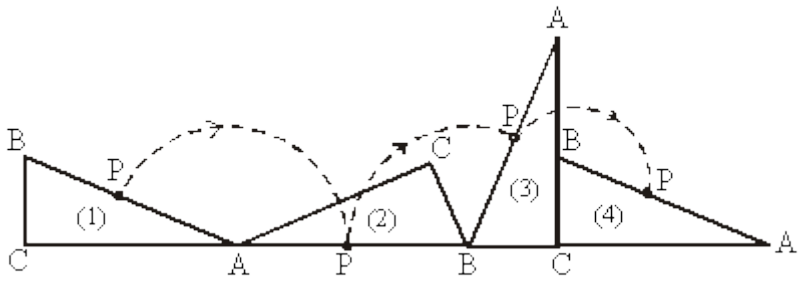
5. A right-angled triangle (1) is placed with one side lying along a straight line.

It is rotated about point A into position (2).

It is then rotated about point B into position (3).

Finally, it is rotated about point C into position (4).

Given that AP = BP = CP = 10 cm, find the total length of the path traced out by point P. (Take $\pi = 3.14$.)



6. Figure 1 shows a street network where *A, B, ..., I* are junctions. We observe that it takes <u>at most</u> 4 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 4 steps.

$$A \xrightarrow{(1)} B \xrightarrow{(2)} E \xrightarrow{(3)} H \xrightarrow{(4)} I$$

The street network is now converted to a one-way traffic system as shown in Figure 2. In this one-way traffic system, it takes <u>at most</u> 6 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 6 steps .

$$A \xrightarrow{(1)} D \xrightarrow{(2)} E \xrightarrow{(3)} B \xrightarrow{(4)} C \xrightarrow{(5)} F \xrightarrow{(6)} I$$

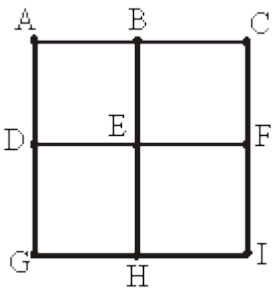


Figure 1

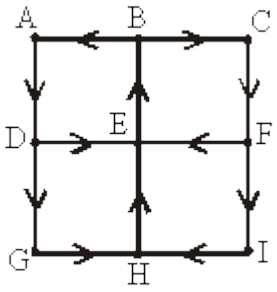


Figure 2

In Figure 3, design a one-way traffic system so that it takes **at most** 5 steps to travel between any two junctions.

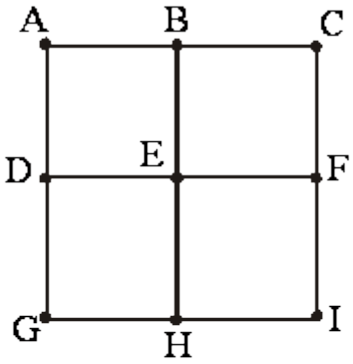


Figure 3

THE END

Singapore Mathematical Olympiad for Primary Schools 2002

Invitation Round – Answers Sheet

Question 1:

Ans: a) 25 b) 2500

Question 2:

Question 3:

Ans:

i) 5^{th} number: 55/89 6^{th} number: 144/233

ii) 2584/4181

iii) 6765/10946

Question 4:

Ans:

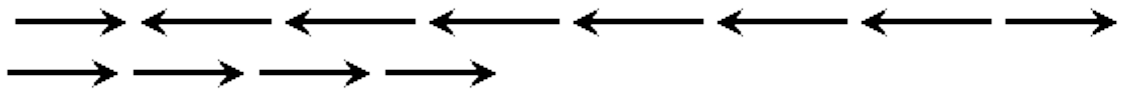
i) 1 : 3

ii) 5 : 11

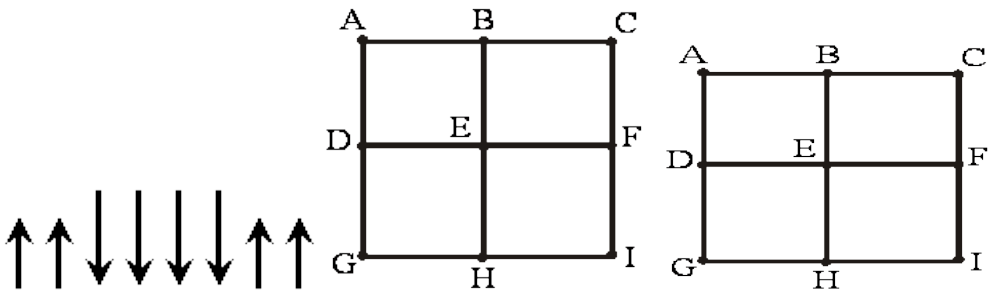
iii) 1 : 2

Question 5:

Ans: 62.8cm



Question 6


A
B
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